Date: October 24, 1998

Def.t. of Math. & Comp. Sci.

First Midterm Exam.

Duration: 75 minutes

Calculators are not allowed

Answer The following questions:

1) a) Evaluate the following limit, if it exists:

$$\lim_{x\to 0} \frac{1+2\sin x - \cos x}{x}$$

(6pts)

b) Use the definition of the derivative to find f'(x), where $f(x) = \sqrt{x^2 + 2}$.

(6pts)

a) State the Intermediate Value Theorem, Show that the equation:

$$2x + \sin x - 1 = 0$$

has at least one real root.

(6pts)

b) Find the vertical and horizontal asymptotes (if any), for the graph of

$$f(x) = \frac{x^2 + x}{(x+1)\sqrt{x^2 + 1}}$$

(6pts)

3) a) Find the value of A so that the function f is continuous in $(-\infty, \infty)$, where

$$f(x) = \begin{cases} \frac{x^2 - 1}{x - 1}, & \text{if } x \neq 1, \\ A, & \text{if } x = 1. \end{cases}$$

(7pts)

b) Find the x-coordinate of the point on the graph of $f(x) = \frac{\sin x}{1 + \cos x} - 3x,$

$$f(x) = \frac{\sin x}{1 + \cos x} - 3x,$$

where $x \in [0, \frac{2}{3}\pi]$ at which the tangent line is parallel to the line 3y + 6x - 11 = 0. (6pts)

$$f(x) = \begin{cases} x - x^{\frac{1}{3}} & \text{if } x \le 0, \\ \sqrt{x} & \text{if } x > 0. \end{cases}$$

Does the graph of f have a cusp at the point (0,0)? Explain.

(7pts)

b) Let
$$y = \sqrt{3u^2 + 5u + 1}$$
, and $u = 4 - \sec^2 x - \tan x$.

Find
$$\frac{dy}{dx}$$
 at $x = \frac{\pi}{4}$

(6pts)

(Good Luck)